Truth theories and their strength – project synopsis

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Research project objectives/Research hypothesis

The far-reaching goal of our project is to better understand how various properties of the notion of *truth* influence its *strength*. We would like to apply our results in the debate over deflationism. Proponents of this, widely discussed nowadays, philosophical stance claim that truth is an "innocent" notion and the truth predicate plays in the language a purely expressive role (for example, it enables expressing infinite conjunctions and disjunctions). The former thesis is often explicated by saying that the notion of truth is explanatory *weak* – it does not permit us to explain anything that could not have been explained without it. Hence, it becomes essential to understand which properties of the notion of truth are *strong* and what the notion of *strength* precisely means in this context.

The direct aim of our project is to obtain results on two fine-grained notions of strength, which are used to investigate axiomatic truth theories: model-theoretical strength and relative truth definability. Both notions play an important role in the philosophical debate over deflationism. According to the first measure of strength, the more of possible interpretations (models of the base theory) a theory excludes, the stronger it is. According to the second one, a theory is *strong* if it can define many different truth predicates (it can embrace many different truth theories).

The aforementioned goals correspond to three main tasks of the project:

Task 1: *Analysis of model-theoretical strength of truth theories*

Task 2: *Analysis of truth-definability relations between truth theories*

Task 3: *Providing philosophical interpretation of the results*

Research project methodology

In our research we rely heavily on the methods of mathematical logic. *Axiomatic truth theories* provide us with formal representations of various properties of the notion of truth. These theories are extensions of Peano arithmetic formulated in the arithmetical language enriched with one new unary predicate $T(x)$. In the project axiomatic truth theories are compared with respect to their strength, with the last relation characterized in terms of expansions of models and relative truth definability. In view of this, methods of model theory, proof theory and set theory will be extensively used.

In particular, in the mathematical part of our project, the theory of nonstandard models of arithmetic plays a crucial role. In order to achieve our goals it will be important to understand which model-theoretical properties are imposed by the assumption that a given model expands (with preservation of the universe) to a model of a given theory of truth.

For Task 3, methods of philosophical analysis will be applied. All in all, this is an interdisciplinary research in which mathematical tools are applied in order to give answers to philosophical questions.

Expected impact of the research project

The results will certainly advance our understanding of the relation between various kinds of axioms for the truth predicate and the strength of the resulting theory of truth. Moreover, the results may in turn make the investigated relations (i.e. relative truth definability and model-theoretical strength) more believable as good measures of the *strength* of the notion of truth. Possibly these relations could later be used in investigations concerning other epistemic notions (such as, e.g., believability or knowledge).

Additionally, the results will be interesting not only for philosophers but also for more mathematically-oriented logicians. Thanks to the links between our research and the theory of nonstandard models of arithmetic, we hope to obtain new results on purely model-theoretical notions such as, e.g., recursive saturation. Moreover,

truth theories are often used in estimating the proof-theoretical strength of various other mathematical theories important from the point of view of foundations of mathematics, so a good grasp of their properties might prove fruitful also in this field.

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